Lesson 7: Infinite Decimals

Classwork

Opening Exercises 1–4

1. Write the expanded form of the decimal $0.3765$ using powers of $10$.
2. Write the expanded form of the decimal $0.3333333…$ using powers of $10$.
3. What is an infinite decimal? Give an example.
4. Do you think it is acceptable to write that $1=0.99999…$? Why or why not?

**Example 1**

The number $0.253$ on the number line:



**Example 2**

The number $\frac{5}{6}=0.833333…=0.8\overbar{3}$ on the number line:



Exercises 5–10

1. a. Write the expanded form of the decimal $0.125$ using powers of $10$.
	1. Show on the number line the representation of the decimal $0.125$.

 

* 1. Is the decimal finite or infinite? How do you know?
1. a. Write the expanded form of the decimal $0.3875$ using powers of $10$.
	1. Show on the number line the representation of the decimal $0.3875$.

 

* 1. Is the decimal finite or infinite? How do you know?
1. a. Write the expanded form of the decimal $0.777777… $using powers of $10$.
	1. Show on the number line the representation of the decimal $0.777777…$.

 

* 1. Is the decimal finite or infinite? How do you know?
1. a. Write the expanded form of the decimal $0.\overbar{45} $using powers of $10$.
	1. Show on the number line the representation of the decimal $0.\overbar{45}$.



* 1. Is the decimal finite or infinite? How do you know?
1. Order the following numbers from least to greatest: $2.121212$, $2.1$, $2.2$, and $2.\overbar{12}.$
2. Explain how you knew which order to put the numbers in.

Lesson Summary

An infinite decimal is a decimal whose expanded form and number line representation are infinite.

Example:
The expanded form of the decimal $0.83333…$ is $0.8\overbar{3}=\frac{8}{10}+\frac{3}{10^{2}}+\frac{3}{10^{3}}+\frac{3}{10^{4}}+…$

The number is represented on the number line shown below. Each new line is a magnification of the interval shown above it. For example, the first line is the unit from $0$ to $1$ divided into $10$ equal parts, or tenths. The second line is the interval from$ 0.8 $to $0.9$ divided into ten equal parts, or hundredths. The third line is the interval from $0.83$ to $0.84$ divided into ten equal parts, or thousandths, and so on.



With each new line we are representing an increasingly smaller value of the number, so small that the amount approaches a value of 0. Consider the $20$th line of the picture above. We would be adding $\frac{3}{10^{20}}$ to the value of the number, which is $0.00000000000000000003$. It should be clear that $\frac{3}{10^{20}}$ is a very small number and is fairly close to a value of $0$.

This reasoning is what we use to explain why the value of the infinite decimal $0.\overbar{9}$ is $1$.

Problem Set



1. a. Write the expanded form of the decimal $0.625$ using powers

of $10$.

* 1. Show the representation of the decimal $0.625$ on the number line.
	2. Is the decimal finite or infinite? How do you know?
1.  a. Write the expanded form of the decimal $0.\overbar{370} $using powers

of $10$.

* 1. Show on the number line the representation of the decimal $0.370370…$
	2. Is the decimal finite or infinite? How do you know?
1. Which is a more accurate representation of the number $\frac{2}{3}:$ $0.6666$ or $0.\overbar{6}$? Explain. Which would you prefer to compute with?
2. Explain why we shorten infinite decimals to finite decimals to perform operations. Explain the effect of shortening an infinite decimal on our answers.
3. A classmate missed the discussion about why $0.\overbar{9}=1.$ Convince your classmate that this equality is true.
4. Explain why $0.3333<0.33333.$