Lesson 5: Negative Exponents and the Laws of Exponents

Classwork

**Definition:** For any positive number $x $and for any positive integer $n$, we define $x^{-n}=$ $\frac{1}{x^{n}}$.

Note that this definition of negative exponents says $x^{-1}$ is just the reciprocal, $\frac{1}{x}$, of $x$.

As a consequence of the definition, for a positive $x $and all *integers* $b$*,* we get

$x^{-b}=\frac{1}{x^{b}}$.

Exercise 1

Verify the general statement $x^{-b}=$ $\frac{1}{x^{b}}$ for $x=3$ and $b=-5$.

Exercise 2

What is the value of $\left(3×10^{-2}\right)$?

Exercise 3

What is the value of $\left(3×10^{-5}\right)$?

Exercise 4

Write the complete expanded form of the decimal $4.728$ in exponential notation.

For Exercises 5–10, write an equivalent expression, in exponential notation, to the one given and simplify as much as possible.

|  |  |
| --- | --- |
| Exercise 5$$5^{-3}=$$ | Exercise 6$$\frac{1}{8^{9}}=$$ |
| Exercise 7$$3∙2^{-4}=$$ | Exercise 8Let $x$ be a nonzero number.$$x^{-3}=$$ |
| Exercise 9Let $x$ be a nonzero number.$$\frac{1}{x^{9}}=$$ | Exercise 10Let $x, y$ be two nonzero numbers.$$xy^{-4}=$$ |

We accept that for positive numbers $x$, $y$ and all integers $a$ and $b$,

$x^{a}∙x^{b}=x^{a+b}$

$ \left(x^{b}\right)^{a}=x^{ab}$

$ \left(xy\right)^{a}=x^{a}y^{a}$.

We claim

$\frac{x^{a}}{x^{b}}$ $=x^{a-b}$ for all *integers* $a$, $b$.

$\left(\frac{x}{y}\right)^{a}=\frac{x^{a}}{y^{a}}$ for any *integer* $a$*.*

|  |  |
| --- | --- |
| Exercise 11$$\frac{19^{2}}{19^{5}}=$$ | Exercise 12$$\frac{17^{16}}{17^{-3}}=$$ |

Exercise 13

If we let $b=-1$ in (11), $a$be any integer, and$ y$ be any positive number, what do we get?

Exercise 14

Show directly that $\left(\frac{7}{5}\right)^{-4}=\frac{7^{-4}}{5^{-4}}$.

Problem Set

1. Compute: $3^{3}×3^{2}×3^{1}×3^{0}×3^{-1}×3^{-2}=$

Compute: $5^{2}×5^{10}×5^{8}×5^{0}×5^{-10}×5^{-8}=$

Compute for a nonzero number, $a$: $a^{m}×a^{n}×a^{l}×a^{-n}×a^{-m}×a^{-l}×a^{0}=$

1. Without using (10), show directly that $\left(17.6^{-1}\right)^{8}=17.6^{-8}$.

1. Without using (10), show (prove) that for any whole number $n$ and any positive number $y$, $\left(y^{-1}\right)^{n}=y^{-n}$.
2. Show directly without using (13) that $\frac{2.8^{-5}}{2.8^{7}}$ $=2.8^{-12}$.